Government Women College of engineering, Ajmer

Mechanical Engineering Department

Mid Term Paper I

Semester:-VI **22/02/2018**

Subject: - Design of Machine Elements **[Marks-20]**

Question 1

(a) What is stress concentration? [1]
(b) Discuss any two causes of stress concentration? [2]
(c) Discuss any two methods to reduce stress concentration? [2]

Question 2 [10]

(a) The section of a steel shaft is shown in Figure 1. The shaft is machined by a turning process. The section at XX is subjected to a constant bending moment of 500 KN-m. The shaft material has ultimate tensile strength of 500 MN/m^2, yield point of 350 MN/m^2 and endurance limit in bending for a 7.5mm diameter specimen of 210 MN/m^2. The notch sensitivity factor can be taken as 0.8. The theoretical stress concentration factor may be interpolated from following tabulated values.

$\frac{r_f}{d}$	0.025	0.05	0.1
K_t	2.6	2.05	1.66

Where r_f the fillet radius and d is the shaft diameter. The reliability is 90%.
Determine the life of the shaft.

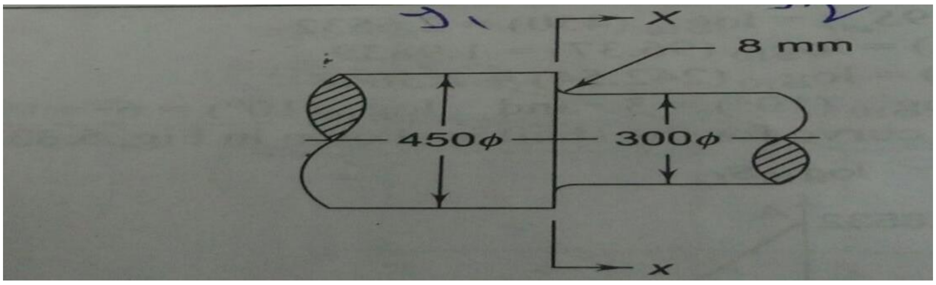


Figure 1

Or

(b) A cantilever beam made of cold drawn steel 4OC8 (S_{ut}=600 N/mm^2 and S_{yt}=380 N/mm^2) is shown in Figure 2. The force P acting at the free end varies from -50 to +150 N. The expected reliability is 90% and the factor of safety is 2. The notch sensitivity factor at the fillet is 0.9. Determine the diameter 'd' of the beam at the fillet cross section.

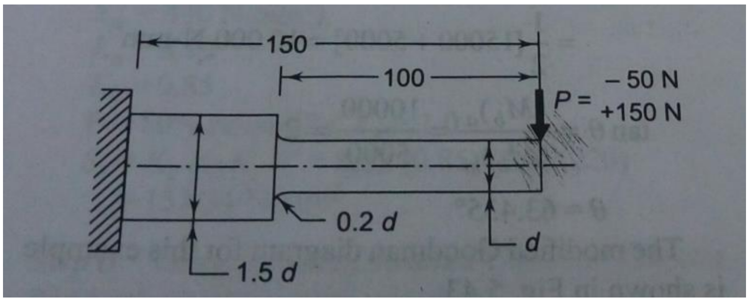


Figure 2

Question 3 [3]

Pair of helical gears are to transmit 15 KW. The teeth are 20 degree stub in diametral plane and have a helix angle of 45 degree. The pinion runs at 10000 rpm and has 80mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 Mpa; determine a suitable module and face width.

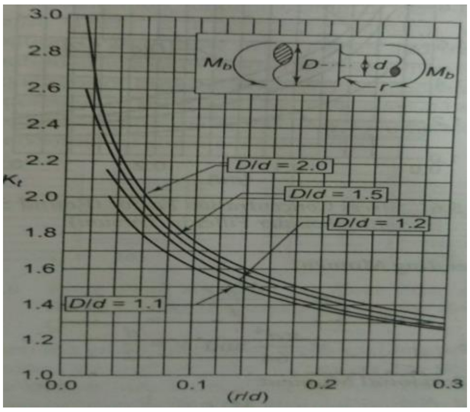
Question 4 [2]

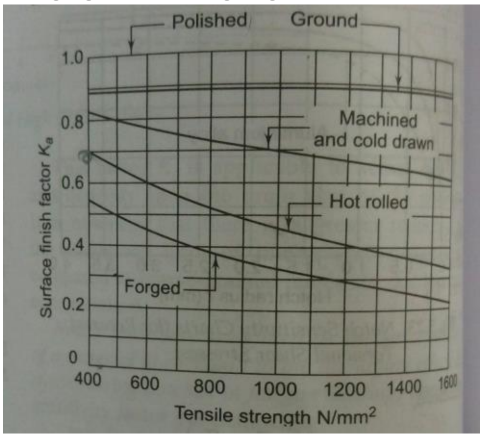
(a) Explain Following (**Any 2**)
- Pressure Angle
- Backlash
- Line of action

Or

(b) Discuss the methods to avoid interference.

Refer the following figures for designing





Diameter (d) (mm)	K_b
$d \le 7.5$	1.00
$7.5 < d \le 50$	0.85
$d > 50$	0.75

Reliability R (%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

The value of velocity factor (C_v) may be taken as follows :

$$C_v = \frac{6}{6+v}, \text{ for peripheral velocities from 5 m / s to 10 m / s.}$$

$$= \frac{15}{15+v}, \text{ for peripheral velocities from 10 m / s to 20 m / s.}$$

$$= \frac{0.75}{0.75+\sqrt{v}}, \text{ for peripheral velocities greater than 20 m / s.}$$

$$= \frac{0.75}{1+v} + 0.25, \text{ for non-metallic gears.}$$

$T_E = T / \cos^3 \alpha$

T = Actual number of teeth on a helical gear, and

α = Helix angle.

$$y = 0.124 - \frac{0.684}{T}, \text{ for } 14\tfrac{1}{2}^\circ \text{ composite and full depth involute system.}$$

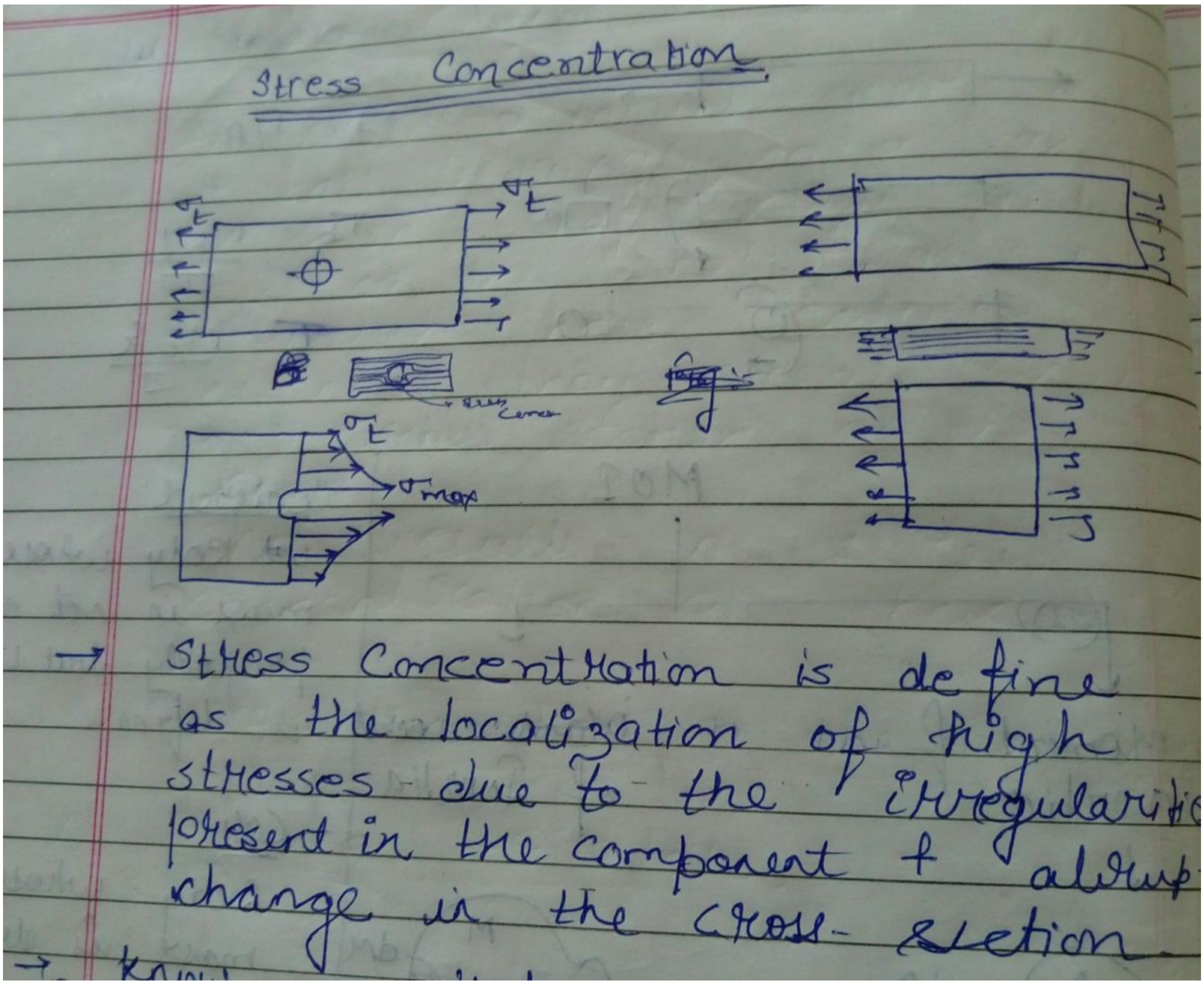
$$= 0.154 - \frac{0.912}{T}, \text{ for } 20^\circ \text{ full depth involute system.}$$

$$= 0.175 - \frac{0.841}{T}, \text{ for } 20^\circ \text{ stub system.}$$

SOLUTIONS

Ans1.

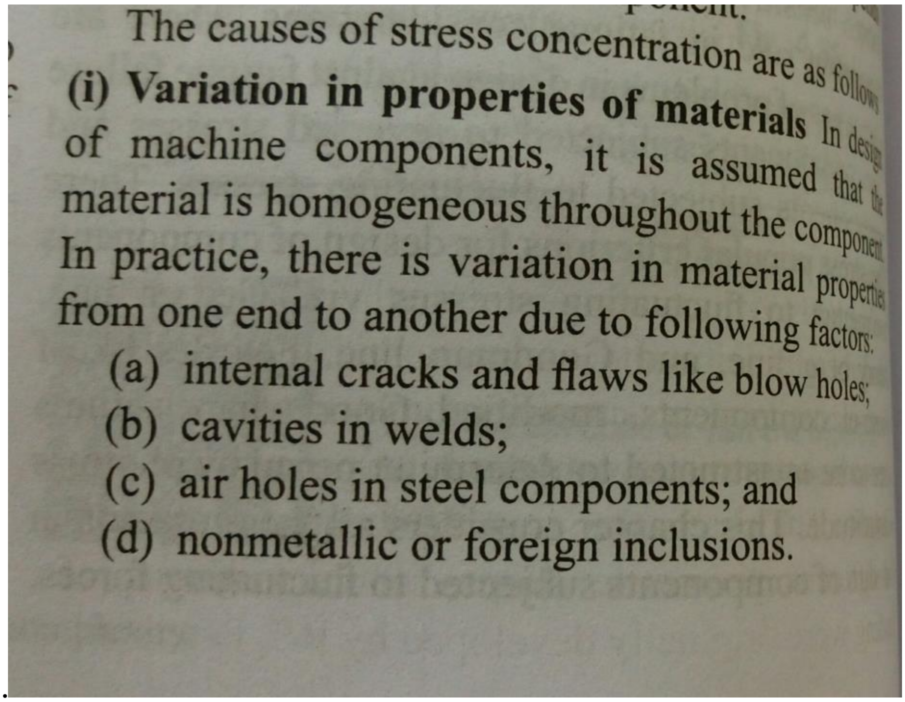
a.



Stress Concentration.

→ Stress Concentration is define as the localization of high stresses due to the irregulariti present in the component & abrup change in the cross-section.

b.



The causes of stress concentration are as follows

(i) Variation in properties of materials In design of machine components, it is assumed that the material is homogeneous throughout the component. In practice, there is variation in material properties from one end to another due to following factors:

(a) internal cracks and flaws like blow holes;

(b) cavities in welds;

(c) air holes in steel components; and

(d) nonmetallic or foreign inclusions.

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These variations act as discontinuities in the component and cause stress concentration.

(ii) Load application Machine components are subjected to forces. These forces act either at a point or over a small area on the component. Since the area is small, the pressure at these points is excessive. This results in stress concentration. The examples of these load applications are as follows:

(a) Contact between the meshing teeth of the driving and the driven gear

(b) Contact between the cam and the follower

(c) Contact between the balls and the races of ball bearing

(d) Contact between the rail and the wheel

(e) Contact between the crane hook and the chain.

In all these cases, the concentrated load is applied over a very small area resulting in stress concentration.

(iii) Abrupt changes in section In order to mount gears, sprockets, pulleys and ball bearings on a transmission shaft, steps are cut on the shaft and shoulders are provided from assembly considerations. Although, these features are essential, they create change of the cross-section of the shaft. This results in stress concentration at these cross-sections.

(iv) Discontinuities in the component Certain features of machine components such as oil holes or oil grooves, keyways and splines, and screw threads result in discontinuities in the cross-section of the component. There is stress concentration in the vicinity of these discontinuities.

(v) Machining scratches Machining scratches, stamp mark or inspection mark are surface irregularities, which cause stress concentration.

c.

In practice, reduction of stress concentration is achieved by following methods:

(i) Additional notches and holes in tension member A flat plate with a V-notch subjected to tensile force is shown in Fig. 5.9 (a). It is observed that a single notch results in a high degree of stress concentration. The severity of stress concentration is reduced by three methods: (a) use of multiple notches; (b) drilling additional holes; and (c) removal of undesired material. These methods are illustrated in Fig. 5.9 (b), (c), and (d) respectively. The method of removing undesired material is called the principle of minimization of the material. In these three methods, the sharp bending of force flow line is reduced and it follows a smooth curve.

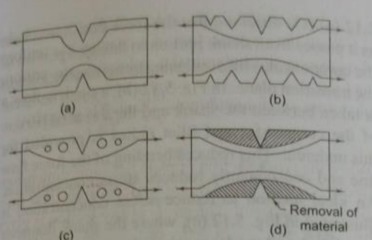


Fig. 5.9 Reduction of stress concentration due to V-notch (a) original notch (b) multiple notches (c) drilled holes (d) removal of undesirable material

(ii) Fillet radius, undercutting and notch for member in bending A bar of circular cross-section with a shoulder and subjected to bending moment is shown in Fig. 5.10 (a). Ball bearings, gears or pulleys are seated against this shoulder. The shoulder creates a change in cross-section of the shaft, which results in stress concentration. There are three methods to reduce stress concentration at the base of this shoulder. Fig. 5.10 (b) shows the shoulder with a fillet radius r. This results in gradual transition from small diameter to large diameter. The fillet radius should be as large as possible in order to reduce stress concentration. In practice, fillet radius is limited by the design of mating components. The fillet radius can be increased by undercutting the shoulder as illustrated in Fig. 5.10 (c). A notch results in stress concentration. Surprisingly, cutting an additional notch is an effective way to reduce stress concentration. This is illustrated in Fig. 5.10 (d).

(iii) Drilling additional holes for shaft A transmission shaft with a keyway is shown in Fig. 5.11 (a). The keyway is a discontinuity and results in stress concentration at the corners of the keyway and reduces torsional shear strength. An empirical relationship developed by H. F. Moore for the ratio 'C' of torsional strength of shaft having a keyway to torsional strength of same sized shaft without keyway is given by

$$C = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) \qquad (5.8)$$

where w and h are width and height dimensions of the keyway respectively and d is the shaft diameter. The four corners of the keyway viz. m_1, m_2, n_1 and n_2 are shown in Fig. 5.11 (c). It has been observed that torsional shear stresses at two points viz. m_1 and m_2 are negligibly small in practice and theoretically equal to zero. On the other hand, the torsional shear stresses at two points viz. n_1 and n_2 are excessive and theoretically infinite which means even a small torque will produce permanent set at these points. Rounding

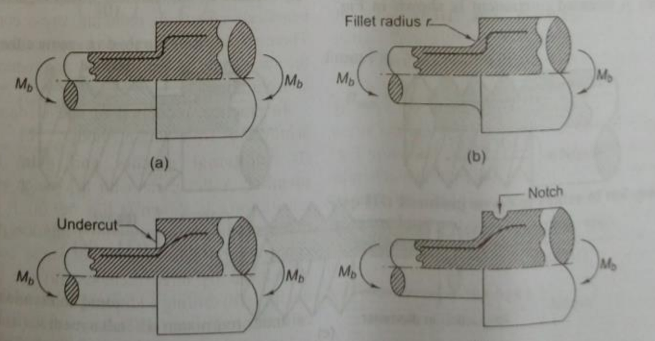
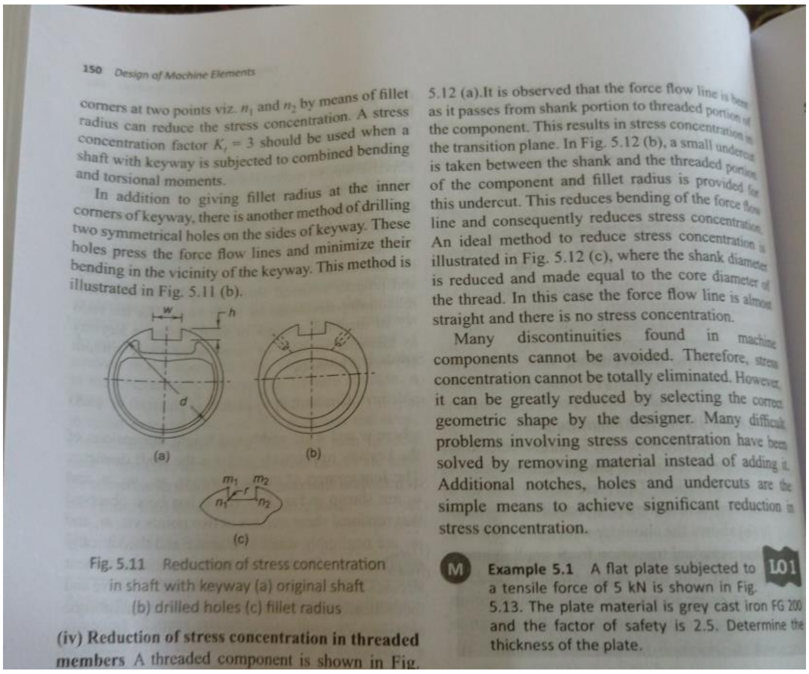


Fig. 5.10 Reduction of stress concentration due to abrupt change in cross-section (a) original component (b) fillet radius (c) undercutting (d) addition of notch



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corners at two points viz. n_1 and n_2 by means of fillet radius can reduce the stress concentration. A stress concentration factor $K_t = 3$ should be used when a shaft with keyway is subjected to combined bending and torsional moments.

In addition to giving fillet radius at the inner corners of keyway, there is another method of drilling two symmetrical holes on the sides of keyway. These holes press the force flow lines and minimize their bending in the vicinity of the keyway. This method is illustrated in Fig. 5.11 (b).

Fig. 5.11 Reduction of stress concentration in shaft with keyway (a) original shaft (b) drilled holes (c) fillet radius

(iv) Reduction of stress concentration in threaded members A threaded component is shown in Fig. 5.12 (a).It is observed that the force flow line is bent as it passes from shank portion to threaded portion of the component. This results in stress concentration in the transition plane. In Fig. 5.12 (b), a small undercut is taken between the shank and the threaded portion of the component and fillet radius is provided for this undercut. This reduces bending of the force flow line and consequently reduces stress concentration. An ideal method to reduce stress concentration is illustrated in Fig. 5.12 (c), where the shank diameter is reduced and made equal to the core diameter of the thread. In this case the force flow line is almost straight and there is no stress concentration.

Many discontinuities found in machine components cannot be avoided. Therefore, stress concentration cannot be totally eliminated. However, it can be greatly reduced by selecting the correct geometric shape by the designer. Many difficult problems involving stress concentration have been solved by removing material instead of adding it. Additional notches, holes and undercuts are the simple means to achieve significant reduction in stress concentration.

M **Example 5.1** A flat plate subjected to a tensile force of 5 kN is shown in Fig. 5.13. The plate material is grey cast iron FG 200 and the factor of safety is 2.5. Determine the thickness of the plate. LO1

Ans 2.

a

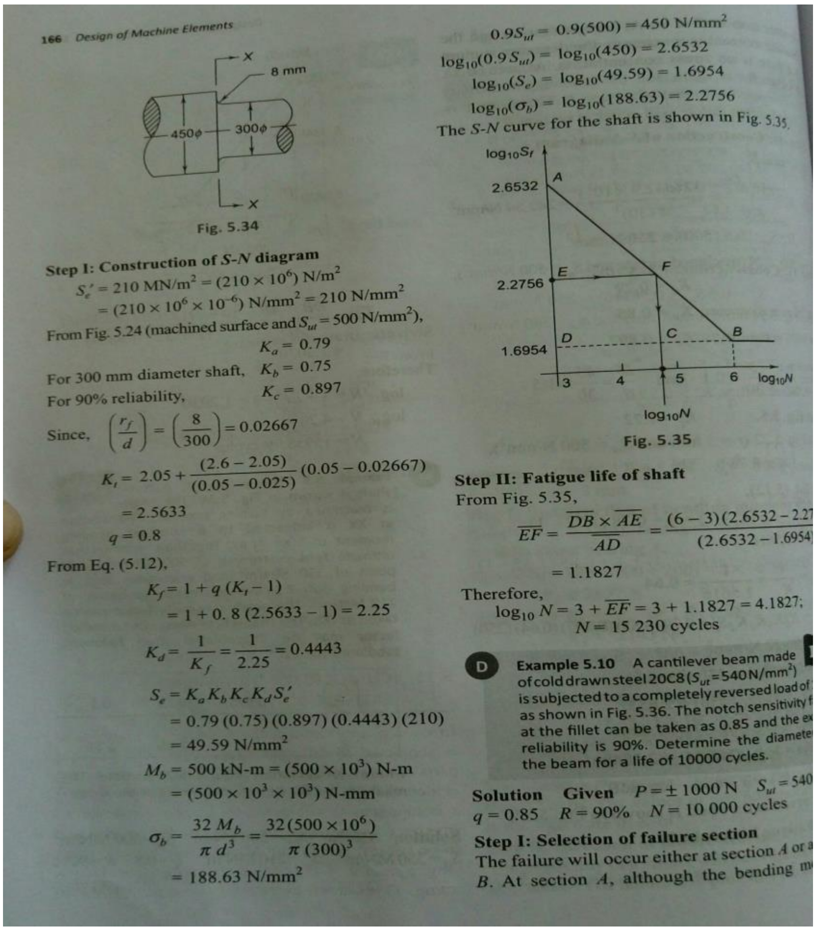

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Fig. 5.34

Step I: Construction of *S-N* diagram

$S_e' = 210\ \text{MN/m}^2 = (210 \times 10^6)\ \text{N/m}^2$

$= (210 \times 10^6 \times 10^{-6})\ \text{N/mm}^2 = 210\ \text{N/mm}^2$

From Fig. 5.24 (machined surface and $S_{ut} = 500\ \text{N/mm}^2$),

$K_a = 0.79$

For 300 mm diameter shaft, $K_b = 0.75$

For 90% reliability, $K_c = 0.897$

Since, $\left(\frac{r_f}{d}\right) = \left(\frac{8}{300}\right) = 0.02667$

$$K_t = 2.05 + \frac{(2.6 - 2.05)}{(0.05 - 0.025)}(0.05 - 0.02667)$$

$= 2.5633$

$q = 0.8$

From Eq. (5.12),

$K_f = 1 + q(K_t - 1)$

$= 1 + 0.8(2.5633 - 1) = 2.25$

$$K_d = \frac{1}{K_f} = \frac{1}{2.25} = 0.4443$$

$S_e = K_a K_b K_c K_d S_e'$

$= 0.79(0.75)(0.897)(0.4443)(210)$

$= 49.59\ \text{N/mm}^2$

$M_b = 500\ \text{kN-m} = (500 \times 10^3)\ \text{N-m}$

$= (500 \times 10^3 \times 10^3)\ \text{N-mm}$

$$\sigma_b = \frac{32 M_b}{\pi d^3} = \frac{32(500 \times 10^6)}{\pi (300)^3}$$

$= 188.63\ \text{N/mm}^2$

$0.9S_{ut} = 0.9(500) = 450\ \text{N/mm}^2$

$\log_{10}(0.9S_{ut}) = \log_{10}(450) = 2.6532$

$\log_{10}(S_e) = \log_{10}(49.59) = 1.6954$

$\log_{10}(\sigma_b) = \log_{10}(188.63) = 2.2756$

The *S-N* curve for the shaft is shown in Fig. 5.35.

Fig. 5.35

Step II: Fatigue life of shaft

From Fig. 5.35,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.6532 - 2.27}{(2.6532 - 1.6954)}$$

$= 1.1827$

Therefore,

$\log_{10} N = 3 + \overline{EF} = 3 + 1.1827 = 4.1827;$

$N = 15\ 230$ cycles

D **Example 5.10** A cantilever beam made of cold drawn steel 20C8 ($S_{ut} = 540\ \text{N/mm}^2$) is subjected to a completely reversed load of as shown in Fig. 5.36. The notch sensitivity f at the fillet can be taken as 0.85 and the ex reliability is 90%. Determine the diamete the beam for a life of 10000 cycles.

Solution Given $P = \pm 1000$ N $S_{ut} = 540$ $q = 0.85$ $R = 90\%$ $N = 10\ 000$ cycles

Step I: Selection of failure section

The failure will occur either at section *A* or a *B*. At section *A*, although the bending m

b.

Fig. 5.42

Solution Given $P = -50$ N to $+150$ N
$S_{ut} = 600$ N/mm^2 $S_{yt} = 380$ N/mm^2 $R = 90\%$
$(fs) = 2$ $q = 0.9$

Step I: Endurance limit stress for cantilever beam

$$S_e' = 0.5\, S_{ut} = 0.5(600) = 300 \text{ N/mm}^2$$

From Fig. 5.24 (cold drawn steel and $S_{ut} = 600$ N/mm^2),

$$K_a = 0.77$$

Assuming $7.5 < d < 50$ mm

$$K_b = 0.85$$

For 90% reliability, $K_c = 0.897$

Since, $\frac{r}{d} = 0.2$ and $\frac{D}{d} = 1.5$

From Fig. 5.5, $K_t = 1.44$

From Eq. (5.12), $K_f = 1 + q\,(K_t - 1)$

$$= 1 + 0.9\,(1.44 - 1) = 1.396$$

$$K_d = \frac{1}{K_f} = \frac{1}{1.396} = 0.716$$

$$S_e = K_a K_b K_c K_d S_e'$$

$$= 0.77\,(0.85)\,(0.897)\,(0.716)\,(300)$$

$$= 126.1 \text{ N/mm}^2$$

Step II: Construction of modified Goodman diagram

At the fillet cross-section,

$$(M_b)_{max} = 150 \times 100 = 15\,000 \text{ N-mm}$$

$$(M_b)_{min} = -50 \times 100 = -5000 \text{ N-mm}$$

$$(M_b)_m = \frac{1}{2}[(M_b)_{max} + (M_b)_{min}]$$

$$= \frac{1}{2}[15000 - 5000] = 5000 \text{ N-mm}$$

$$(M_b)_a = \frac{1}{2}[(M_b)_{max} - (M_b)_{min}]$$

$$= \frac{1}{2}[15000 + 5000] = 10\,000 \text{ N-mm}$$

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} = \frac{10000}{5000} = 2$$

$$\theta = 63.435°$$

The modified Goodman diagram for this example is shown in Fig. 5.43.

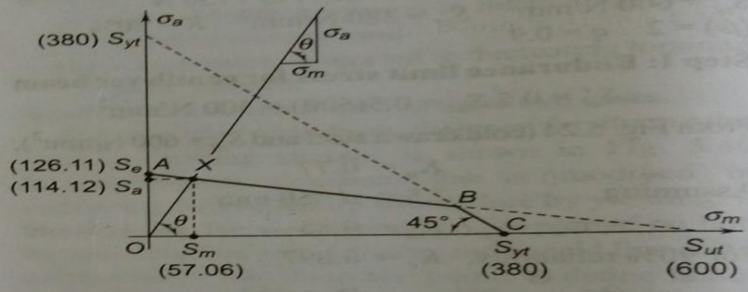

Fig. 5.43

Step III: Permissible stress amplitude

Refer to Fig. 5.43. The coordinates of point X are determined by solving the following two equations simultaneously.

(i) Equation of line AB

$$\frac{S_a}{126.11} + \frac{S_m}{600} = 1 \qquad \text{(a)}$$

(ii) Equation of line OX

$$\frac{S_a}{S_m} = \tan\theta = 2 \qquad \text{(b)}$$

Solving the two equations,

$$S_a = 114.12 \text{ N/mm}^2$$

and $$S_m = 57.06 \text{ N/mm}^2$$

Step IV: Diameter of beam

Since $\sigma_a = \dfrac{S_a}{(fs)}$ $\quad\therefore\quad$ $\dfrac{32\,(M_b)_a}{\pi d^3} = \dfrac{S_a}{(fs)}$

$$\frac{32\,(10\,000)}{\pi d^3} = \frac{114.12}{2}$$

$$d = 12.13 \text{ mm}$$

Ans 3.

Since both the pinion and gear are made of the same material (*i.e.* cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion.

We know that the torque transmitted by the pinion,

$$T = \frac{P \times 60}{2\,\pi\,N_P} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 10000} = 14.32 \text{ N-m}$$

∴ *Tangential tooth load on the pinion,

$$W_T = \frac{T}{D_P / 2} = \frac{14.32}{0.08/2} = 358 \text{ N}$$

We know that number of teeth on the pinion,

$$T_P = D_P / m = 80 / m$$

and formative or equivalent number of teeth for the pinion,

$$T_E = \frac{T_P}{\cos^3 \alpha} = \frac{80/m}{\cos^3 45°} = \frac{80/m}{(0.707)^3} = \frac{226.4}{m}$$

∴ Tooth form factor for the pinion for 20° stub teeth,

$$y'_P = 0.175 - \frac{0.841}{T_E} = 0.175 - \frac{0.841}{226.4/m} = 0.175 - 0.0037\,m$$

We know that peripheral velocity,

$$v = \frac{\pi\,D_P . N_P}{60} = \frac{\pi \times 0.08 \times 10000}{60} = 42 \text{ m/s}$$

∴ Velocity factor,

$$C_v = \frac{0.75}{0.75 + \sqrt{v}} = \frac{0.75}{0.75 + \sqrt{42}} = 0.104 \quad \text{...(∵ } v \text{ is greater than 20 m/s)}$$

Since the maximum face width (b) for helical gears may be taken as 12.5 m to 20 m, where m is the module, therefore let us take

$$b = 12.5\,m$$

We know that the tangential tooth load (W_T),

$$358 = (\sigma_{OP} . C_v)\, b.\pi\, m.y'_P$$

$$= (100 \times 0.104)\ 12.5\,m \times \pi\,m\,(0.175 - 0.0037\,m)$$

$$= 409\,m^2\,(0.175 - 0.0037\,m) = 72\,m^2 - 1.5\,m^3$$

Solving this expression by hit and trial method, we find that

m = 2.3 say 2.5 mm **Ans.**

and face width, $b = 12.5\,m = 12.5 \times 2.5 = 31.25$ say 32 mm **Ans.**

Ans 4.

a] Line of Action :-
The force, which the driving tooth exerts on the driven tooth, is along a line from the pitch point to the point of contact of the two teeth. This line is also the common normal at the point of contact of the mating gears & is known as the line of action or the pressure line.

b] Backlash :
Clearance between mating gear teeth, is built into speed reducers to let the gears mesh without binding & to provide space for a film of lubricating oil b/w the teeth. This prevent overheating & tooth damage

c) Pressure Angle(ϕ) :
Angle b/w the pressure line & the common tangent to the pitch circles is known as pressure Angle.