Govt. Women Engineering College, Ajmer

B. Tech II Sem, Ist Mid Term Test: March 2018,

Engineering Mechanics

Time: 1 Hour **Max. Marks: 20**

Q.1. **Attempt all the questions. Draw labeled diagrams wherever necessary.**

(a). Write the expression of the parallelogram law of forces. **(2)**

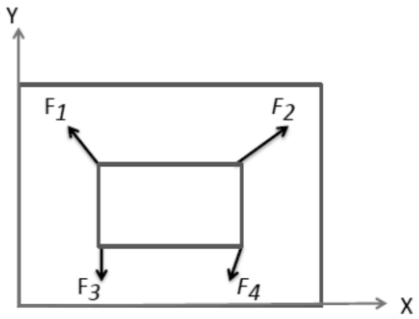
Ans: Resultant force, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2

(b). Sketch a non-concurrent non parallel-coplanar force system. **(2)**

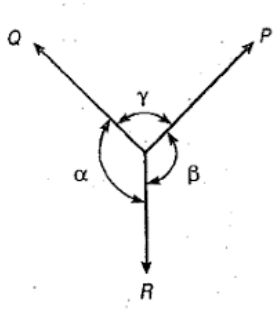
Ans:



(c). State the Lami's theorem. **(2)**

Ans: It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two forces." Mathematically we can express,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



(d). Define the moment of force. **(2)**

Ans: It is the measure of tendency of a force to cause a body to rotate about a specific point or axis. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force. Mathematically;

$$M = r \times F$$

F = Force acting on the body, and

r = Perpendicular distance between the point, about which the moment is required and the line of action of the force

(e). Write the conditions for the equilibrium of a body. **(2)**

Ans: conditions for the equilibrium;

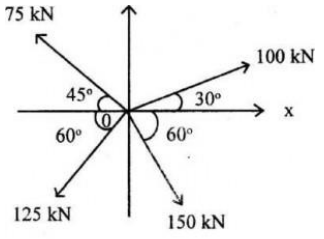
1. The vector sum of all the forces acting on the body must be zero.

$$\sum F = 0$$

2. The body in the equilibrium cannot have a tendency to rotate.

$$\sum M = 0$$

Q.2. Determine the resultant of the co-planar system of concurrent forces as shown in fig. **(5)**



Ans: Sum of horizontal components of all the forces:

$$\sum Fx = \left(100\cos 30^o - 75\cos 45^o - 125\cos 60^o + 150\cos 60^o\right)\text{kN}$$

$$= \left(100 \times \frac{\sqrt{3}}{2} - 75 \times \frac{1}{\sqrt{2}} - 125 \times \frac{1}{2} + 150 \times \frac{1}{2}\right)\text{kN}$$

$$= 46.0695 \text{ kN}$$

Sum of vertical components of all the forces:

$$\sum Fy = (100\sin 30^o + 75\sin 45^o - 125\sin 60^o - 150\sin 60^o)\text{kN}$$

$$= \left(100 \times \frac{1}{2} + 75 \times \frac{1}{\sqrt{2}} - 125 \times \frac{\sqrt{3}}{2} - 150 \times \frac{\sqrt{3}}{2}\right)\text{kN}$$

$$= -135.124\,\text{kN}$$

Magnitude of the resultant force is given by

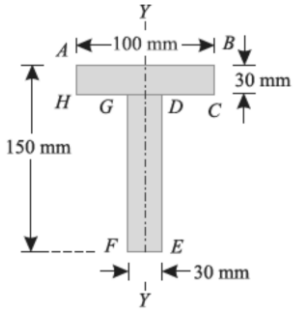
$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$$

Resultant force $= \sqrt{(46.0695)^2 + (-135.124)^2}$ kN

$= 142.762$ kN

OR

Q.2. Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section as shown in fig. **(5)**



Ans: As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles *ABCH* and *DEFG* as shown in Fig.

Let bottom of the web *FE* be the axis of reference.

(i) Rectangle ABCH

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

and $$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(ii) Rectangle DEFG

$$a_2 = 120 \times 30 = 3600 \text{ mm}^2$$

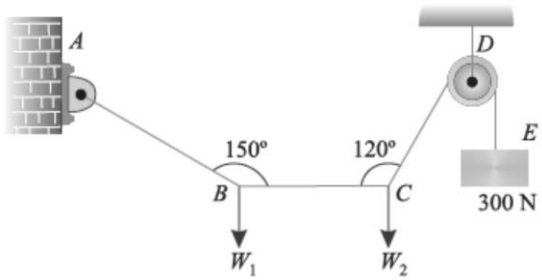
and $$y_2 = \frac{120}{2} = 60 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange *FE*,

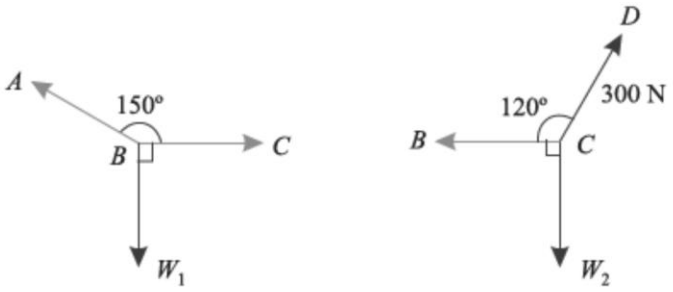
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} \text{ mm}$$

$$= 94.1 \text{ mm} \quad \textbf{Ans.}$$

Q.3. (a) Determine the tension in string AB, BC, and CD. (b) Find the value of weight W_1 and W_2. **(3+2)**



Ans: For the sake of convenience, let us split up the string *ABCD* into two parts. The system of forces at joints *B* and *C* is shown in Fig. (*a*) and (*b*).



(*a*) Joint *B* (*b*) Joint *C*

(*i*) *Tensions is the portion AB, BC and CD of the string*

Let T_{AB} = Tension in the portion *AB*, and

T_{BC} = Tension in the portion *BC*,

We know that tension in the portion *CD* of the string.

$$T_{CD} = T_{DE} = 300 \text{ N} \quad \textbf{Ans.}$$

Applying Lami's equation at C,

$$\frac{T_{BC}}{\sin 150^\circ} = \frac{W_2}{\sin 120^\circ} = \frac{300}{\sin 90^\circ}$$

$$\frac{T_{BC}}{\sin 30^\circ} = \frac{W_2}{\sin 60^\circ} = \frac{300}{1} \qquad \ldots[\because \sin(180^\circ - \theta) = \sin\theta]$$

$\therefore$ $T_{BC} = 300 \sin 30^\circ = 300 \times 0.5 = 150 \text{ N}$ **Ans.**

and $W_2 = 300 \sin 60^\circ = 300 \times 0.866 = 259.8 \text{ N}$

Again applying Lami's equation at *B*,

$$\frac{T_{AB}}{\sin 90^\circ} = \frac{W_1}{\sin 150^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

$$\frac{T_{AB}}{1} = \frac{W_1}{\sin 30^\circ} = \frac{150}{\sin 60^\circ} \qquad \ldots[\because \sin(180^\circ - \theta) = \sin\theta]$$

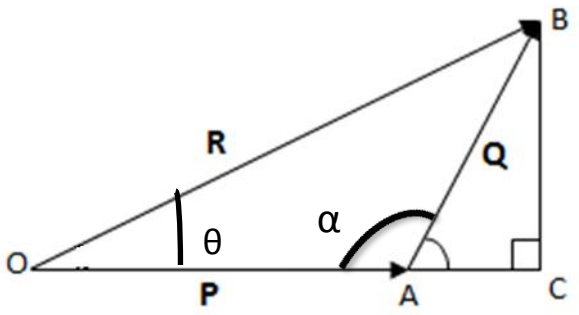
$\therefore$ $T_{AB} = \dfrac{150}{\sin 60^\circ} = \dfrac{150}{0.866} = 173.2 \text{ N}$ **Ans.**

and $W_1 = \dfrac{150 \sin 30^\circ}{\sin 60^\circ} = \dfrac{150 \times 0.5}{0.866} = 86.6 \text{ N}$

OR

Q.3. State and prove the triangle Law of forces. **(2+3)**

Ans: It states that "If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a triangle taken in order, then the third side of the triangle represents the resultant of the forces in magnitude and direction."



PROOF: Consider two forces **P** and **Q** acting on a body and represented both in magnitude and direction by sides OA and AB respectively of a triangle OAB. Let α be the angle between **P** and **Q**. Let **R** be the resultant of vectors **P** and **Q**.

From ΔOBC, we have

$$OB^2 = OC^2 + CB^2$$

$$R^2 = (OA + AC)^2 + CB^2$$

$$= \left(OA + AB\cos(180-\alpha)\right)^2 + \left(AB\sin(180-\alpha)\right)^2 \qquad \because \angle BAC = 180-\alpha$$

$$= \left(P^2 + Q^2\cos(180-\alpha)\right)^2 + \left(Q\sin(180-\alpha)\right)^2$$

$$= P^2 + Q^2\cos^2(180-\alpha) + 2PQ\cos(180-\alpha) + Q^2\sin^2(180-\alpha)$$

$$= P^2 + Q^2 - 2PQ\cos\alpha \qquad \because \cos(180-\alpha) = -\cos\alpha$$

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\alpha} \qquad \text{Resultant of forces P and Q.}$$

$$BC = OB\sin\theta = R\sin\theta = Q\sin(180-\alpha)$$

$$\sin\theta = \frac{Q\sin(180-\alpha)}{R}$$