Govt. Women Engineering college, Ajmer Pg(1)
MODEL ANSWER PAPER
B. Tech II Semester
Sub:- Engineering Mathematics - II

Q1. Investigate the values of λ and μ so that the equations

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

have (i) no solution (ii) unique solution (iii) an infinite number of solutions

Solution :- Argumented matrix for the given system of equations is

$$C = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$$R_2 \to R_2 - \frac{7}{2}R_1, \quad R_3 \to R_3 - R_1$$

$$C \sim \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15/2 & -39/2 & -47/2 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

(i) for no solution $\rho(A) \neq \rho(C)$

$\lambda - 5 = 0, \quad \mu - 9 \neq 0$

$\Rightarrow \lambda = 5$ and $\mu \neq 9$

(ii) for unique solution

$\rho(A) = \rho(C) =$ no. of variables $= 3$

$\therefore \lambda - 5 \neq 0$ and μ can have any value

$\Rightarrow \lambda \neq 5$ and μ can have any value

(iii) for infinite many solution

$\rho(A) = \rho(C) < 3$ which is Possible when

$$\lambda - 5 = 0 \text{ and } \mu - 9 = 0$$
$$\therefore \lambda = 5 \text{ and } \mu = 9$$

Q 2. find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$$

<u>Solution</u> The characteristic equation of A is

$$|A - \lambda I| = \begin{vmatrix} -2-\lambda & 1 & 1 \\ -11 & 4-\lambda & 5 \\ -1 & 1 & -\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)(-4\lambda + \lambda^2 - 5) - 1(11\lambda + 5) + 1(-11 + 4 - \lambda) = 0$$
$$-\lambda^3 + 2\lambda^2 + 13\lambda + 10 - 11\lambda - 5 - 11 + 4 - \lambda = 0$$
$$-\lambda^3 + 2\lambda^2 + \lambda - 2 = 0$$
$$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$$
$$(\lambda - 1)(\lambda^2 - \lambda - 2) = 0$$
$$(\lambda - 1)(\lambda + 1)(\lambda - 2) = 0$$
$$\lambda = 1, -1, 2$$

eigen values are $1, -1, 2$

for $\lambda = 1$

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} -3 & 1 & 1 \\ -11 & 3 & 5 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2 \to R_2 - \frac{11}{3} R_1, \quad R_3 \to R_3 - \frac{1}{3} R_1$$

$$\begin{bmatrix} -3 & 1 & 1 \\ 0 & -2/3 & 4/3 \\ 0 & 2/3 & -4/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} -3 & 1 & 1 \\ 0 & -2/3 & 4/3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + x_2 + x_3 = 0$$

$$-\frac{2}{3}x_2 + \frac{4}{3}x_3 = 0$$

$$x_2 = 2x_3$$

let $x_3 = 1, \Rightarrow x_2 = 2, x_1 = 1$

eigenvector $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

for $\lambda = -1$

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} -1 & 1 & 1 \\ -11 & 5 & 5 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 11R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & -6 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_4 + x_5 + x_6 = 0$$

$$-6x_5 - 6x_6 = 0$$

$$x_5 = -x_6 , \quad x_4 = 0$$

let $x_6 = -1 \Rightarrow x_5 = 1$

$\therefore$ eigenvector $\begin{bmatrix} 0 \\ +1 \\ -1 \end{bmatrix}$

For $\lambda = 2$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} -4 & 1 & 1 \\ -11 & 2 & 5 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - \frac{11}{4} R_1, \quad R_3 \to R_3 - \frac{1}{4} R_1$

$$\begin{bmatrix} -4 & 1 & 1 \\ 0 & -3/4 & 9/4 \\ 0 & 3/4 & -9/4 \end{bmatrix} \begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} -4 & 1 & 1 \\ 0 & -3/4 & 9/4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-4x_7 + x_8 + x_9 = 0$

$-\frac{3}{4} x_8 + \frac{9}{4} x_9 = 0$

$x_8 = 3x_9$

Let $x_9 = 1 \Rightarrow x_8 = 3, \ x_7 = 1$

$\therefore$ Eigen vector is $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

Q3 Solve the differential equation

$$\frac{dy}{dx} + \frac{1}{x}\tan y = \frac{1}{x^2}\frac{\sin^2 y}{\cos y}$$

Solution :- divide by $\frac{\sin^2 y}{\cos y}$ whole eqn

$$\Rightarrow \frac{\cos y}{\sin^2 y}\frac{dy}{dx} + \frac{1}{x}\tan y\,\frac{\cos y}{\sin^2 y} = \frac{1}{x^2}$$

$$\Rightarrow \operatorname{cosec} y \cot y \frac{dy}{dx} + \frac{1}{x}\operatorname{cosec} y = \frac{1}{x^2}$$

$\Rightarrow$ Let $\operatorname{cosec} y = z$

$$-\operatorname{cosec} y \cot y \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow -\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$$

$$\Rightarrow \frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$$

$$\text{IF} \quad e^{\int -\frac{1}{x}dx} = -\log x = \frac{1}{x}$$

Solution is

$$z\left(\frac{1}{x}\right) = \int -\frac{1}{x^2}\left(\frac{1}{x}\right)dx + c$$

$$z\left(\frac{1}{x}\right) = \frac{1}{2x^2} + c$$

$$\frac{\operatorname{cosec} y}{x} = \frac{1}{2x^2} + c$$

Q 4 Solve the differential eqⁿ

$$(D^2-4D+4)y = 8x^2 e^{2x} \sin 2x$$

Solution A·E is $m^2-4m+4=0$

$m = 2, 2$

CF is $(C_1+C_2x)e^{2x}$

$$PI = \frac{1}{D^2-4D+4} 8x^2 e^{2x} \sin 2x$$

$$= 8e^{2x} \frac{1}{(D+2)^2-4(D+2)+4} x^2 \sin 2x$$

$$= 8e^{2x} \frac{1}{D^2+4+4D-4D-8+4} x^2 \sin 2x$$

$$= 8 e^{2x} \frac{1}{D^2} x^2 \sin 2x$$

$$= I \cdot P \text{ of } \left[8e^{2x} \frac{1}{D^2} x^2 e^{i2x}\right]$$

$$= IP \text{ of } \left[8 e^{2x} e^{2ix} \frac{1}{(D+2i)^2} x^2\right]$$

$$= IP \text{ of } \left[8 e^{2x} e^{2ix} \frac{1}{D^2-4+4iD} x^2\right]$$

$$= IP \text{ of } \left[-2 e^{2x} e^{2ix} \frac{1}{\left[1-\left(\frac{D^2+4iD}{4}\right)\right]} x^2\right]$$

$$= IP \text{ of } \left[-2 e^{2x} e^{2ix} \left[1-\left(\frac{D^2+4iD}{4}\right)\right]^{-1} x^2\right]$$

$$= IP \text{ of } \left\{ -2e^{2x} e^{2ix} \left[1 + \left(\frac{D^2+4iD}{4} \right) + \left(\frac{D^2+4iD}{4} \right)^2 + \cdots \right] x^2 \right\}$$

$$= IP \text{ of } \left\{ -2e^{2x} e^{2ix} \left[x^2 + \left(\frac{2+8ix}{4} \right] + \left[\frac{-32}{16} \right] + 0 \right\}$$

$$= IP \text{ of } \left\{ -2e^{2x} e^{2ix} \left[x^2 + \frac{1}{2} + 2ix - 2 \right] \right\}$$

$$= IP \text{ of } \left[-2e^{2x} e^{2ix} \left[x^2 - \frac{3}{2} + 2ix \right] \right]$$

$$= IP \text{ of } \left[-2e^{2x} (\cos 2x + i \sin 2x) \left(x^2 - \frac{3}{2} + 2ix \right) \right]$$

$$= -2e^{2x} \left[2x \cos 2x + x^2 \sin 2x - \frac{3}{2} \sin 2x \right]$$

$\therefore$ Solution is $y = CF + PI$

$$\Rightarrow y = (c_1 + c_2 x) e^{2x} - 2e^{2x} \left[2x \cos 2x + x^2 \sin 2x - \frac{3}{2} \sin 2x \right]$$

Q 5 Solve the differential equation

$$(x^2+y^2+x)\,dx - (2x^2+2y^2-y)\,dy = 0$$

solution:- $(x^2+y^2)\,dx + x\,dx - 2(x^2+y^2)\,dy + y\,dy = 0$

$$= (x^2+y^2)[dx - 2dy] + x\,dx + y\,dy = 0$$

$$= \int dx - \int 2\,dy + \int \frac{x\,dx + y\,dy}{x^2+y^2} = 0$$

Put $x^2+y^2 = t$

$2(x\,dx + y\,dy) = dt$

$$= x - 2y + \int \frac{dt}{2t} = C$$

$$= x - 2y + \frac{1}{2} \log t = C$$

$$= x - 2y + \frac{1}{2} \log(x^2+y^2) = C$$