

Ans. Let p : I try hard.
 q : I have talent
 r : I become a musician
 s : I will be happy

$$\frac{(p \wedge q) \rightarrow r \\ r \rightarrow s}{\sim s \rightarrow (\sim p \vee \sim q)}$$

→ Hy ①
 → Hy ②
 con

- S. No. Step
 1. $(p \wedge q) \rightarrow r$
 2. $r \rightarrow s$
 3. $(p \wedge q) \rightarrow s$

Reason.

Hy ①
 Hy ②
 $p \rightarrow q$ [Hypothetical]
 $q \rightarrow r$ [syllogism]
 $p \rightarrow r$
 Using step ① & ②

4. $\sim s \rightarrow \sim(p \wedge q)$

$p \rightarrow q \equiv \sim q \rightarrow \sim p$
 [contrapositive]
 Using step 3.

5. $\sim s \rightarrow \sim p \vee \sim q$

$\sim(p \wedge q) \equiv \sim p \vee \sim q$
 Using step 4.
 [De Morgan's law]

Hence the argument is valid.

today.

Ans 2. (i) Let p : Today is Wednesday
 q : I have a test today

Converse is $q \rightarrow p$: If I have a test today,
then today is Wednesday.

Inverse is $\sim p \rightarrow \sim q$: If today is not Wednesday,
then I do not have a test today.

Contrapositive: $\sim q \rightarrow \sim p$: If I do not have a test today, then today is not Wednesday.

Ans 3. Vacuous Proof

If the hypothesis p is false then the implication $p \rightarrow q$ is always true by default or it is vacuously true.

Thus if the hypothesis p can be shown to be false then the theorem $p \rightarrow q$ is true by default. Such a proof is called a vacuous proof.

Ex 1. Show that the proposition 'If $1=2$ then $3=4$ ' is true.

Sol" Let $p: 1=2$ and $q: 3=4$

Since here the hypothesis p is false ($\because 1 \neq 2$) so the proposition $p \rightarrow q$ is vacuously true.

Trivial Proof

If the conclusion q is true then the implication $p \rightarrow q$ is always true, irrespective of the truth value of p .

Thus if the conclusion q can be shown to be true then such a proof of the implication $p \rightarrow q$ is called a trivial proof.

Ex. • $P(n)$: If x is a +ve real number and n is any non negative integer, then $(1+x)^n \geq 1+nx$, is true for $n=0$

The conclusion of $P(n)$ is $(1+x)^n \geq 1+nx$

Now for $n=0$, we have

$$(1+x)^0 \geq 1+0 \cdot x$$

$$\text{or} \quad 1 \geq 1$$

which is true.

Thus the proposition $P(n)$ is trivially true when $n=0$.

Ans 4 - Let p : n is not divisible by 3
and q : $n^2 \equiv 1 \pmod{3}$
Then

$$\begin{aligned} p &= p_1 \vee p_2 \\ p_1 &: n \equiv 1 \pmod{3} \\ p_2 &: n \equiv 2 \pmod{3} \end{aligned}$$

Hence, to show that $p \rightarrow q$ it can be shown that
 $p_1 \rightarrow q$ and $p_2 \rightarrow q$

First, let p_1 is true. Then $n \equiv 1 \pmod{3}$, so $n = 3k+1$
for some integer k . Thus

$$\begin{aligned} n^2 &= (3k+1)^2 = 9k^2 + 12k + 1 \\ &= 3(3k^2 + 4k) + 1 \\ &= 3m+1 \end{aligned}$$

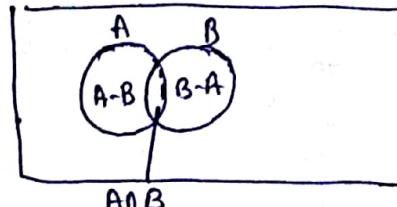
where $m = 3k^2 + 4k + 1$ is an integer

Hence, we have $n^2 \equiv 1 \pmod{3}$ so $p_1 \rightarrow q$ is true.

Since both $p_1 \rightarrow q$ and $p_2 \rightarrow q$ are true, it can be concluded that $(p_1 \vee p_2) \rightarrow q$ is true. Moreover it follows that $p \rightarrow q$ is true.

Auss. We know that by ^{venn} Diagram
 $(A-B) \cup (A \cap B) \cup (B-A) = A \cup B$

and



—①

$A-B$, $A \cap B$ and $B-A$ are pairwise disjoint
 therefore.

Further $n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A)$ —②

$$A = (A-B) \cup (A \cap B)$$

and

$$(A-B) \cap (A \cap B) = \emptyset$$

so

$$n(A) = n(A-B) + n(A \cap B)$$

likewise

$$n(B) = n(A \cap B) + n(B-A)$$

—③

—④

Hence, adding ③ and ④ we have.

$$\begin{aligned} n(A) + n(B) &= [n(A-B) + n(A \cap B) + n(B-A)] + n(A \cap B) \\ &= n(A \cup B) + n(A \cap B) \end{aligned}$$

Thus $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Hence proved

Ans. Let A , B , C denote the set of students who study Hindi, English and German language respectively

Given that

$$|A \cup B \cup C| = 160, |A| = 65, |B| = 45, |C| = 42 \\ |A \cap B| = 20, |B \cap C| = 25, |A \cap C| = 15$$

$$(A \cap B \cap C) = ?$$

By principle of Inclusion-Exclusion, we have

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \\ 160 = 65 + 45 + 42 - 20 - 15 - 25 + |A \cap B \cap C|$$

$$|A \cap B \cap C| = 160 - 152 = 8$$

Hence 8 students studying all the three languages.

GOVT. WOMEN ENGINEERING COLLEGE, AJMER

B.Tech. IV Semester (I Midterm) (CS, IT)

Sub.: DMS

Max. Marks: 20

Time: 1 hr.

Q.1 Check the validity of the Argument:

If I try hard and I have talent, then I will become a musician. If I become a musician then I

Will be happy . Therefore If I will not be happy, then I did not try hard or I do not have talent.

Q.2 Find the Converse, Inverse and Contra positive of the implication:

"If today is Wednesday, then I have a test today".

Q.3 Define Vacuous proof and Trivial proof with Example.

Q.4 Prove the implication "If n is an integer not divisible by 3, then $n^2 \equiv 1 \pmod{3}$ ".

Q.5 Prove, for finite sets A and B , $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

OR

Let 100 of the 120 students of mathematics at a college take at least one of the languages Hindi, English and German. Also, let 65 study Hindi, 45 study English and 42 German if 20 study Hindi and English, 25 study English and German and 15 study Hindi and German. Find the number of students who study all the three languages.