Name of Subject - SPT
B. Tech. IV Sem. CS & I.T.

Model Solutions.

1. Let the events be
B_1 = Production from first Plant.
$B_2 \rightarrow$ " " second plant
$B_3 \rightarrow$ " Third plant
$D \rightarrow$ Defective Output

Given $P(B_1) = \frac{500}{3500} = \frac{1}{7}$ $\quad P(B_2) = \frac{1000}{3500} = \frac{2}{7}$ $\quad P(B_3) = \frac{2000}{3500} = \frac{4}{7}$

and $P\left(\frac{D}{A}\right) = .005$ $\quad P\left(\frac{D}{B}\right) = .008$ $\quad P\left(\frac{D}{C}\right) = .010$

Probability that a pipe selected is defective is

$$P(D) = P(B_1).P\left(\frac{D}{B_1}\right) + P(B_2)P\left(\frac{D}{B_2}\right) + P(B_3).P\left(\frac{D}{B_3}\right)$$

$$= \frac{1}{7} \times .005 + \frac{2}{7} \times .008 + \frac{4}{7} \times .010 = .00871.$$

Also by Baye's Theorem.

$$P\left(\frac{B_1}{D}\right) = \frac{P(B_1).P\left(\frac{D}{B_1}\right)}{P(D)} = \frac{\frac{1}{7} \times .005}{.00871} = .0815$$

$$P\left(\frac{B_2}{D}\right) = \frac{P(B_2).P\left(\frac{D}{B_2}\right)}{P(D)} = \frac{\frac{2}{7} \times .008}{.00871} = .2629$$

$$P\left(\frac{B_3}{D}\right) = \frac{P(B_3).P\left(\frac{D}{B_3}\right)}{P(D)} = \frac{\frac{4}{7} \times .010}{.00871} = .6556.$$

2)

(2) (i) As $f(x)$ is given to be a pdf hence

$$\int_0^\infty f(x)\,dx = 1$$

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx = 1 \Rightarrow 2a = 1, \quad a = 1/2$$

(ii) $$P(X \leq 1.5) = \int_0^1 f(x)\,dx + \int_1^{1.5} f(x)\,dx$$

$$= \int_0^1 ax\,dx + \int_1^{1.5} a\,dx = \frac{a}{2} + \frac{a}{2} = a = 1/2$$

(iii) For $x < 0$ $\quad F(x) = 0$

$0 \leq x \leq 1$ $\quad F(x) = \int_0^x f(x)\,dx = \int_0^x ax\,dx = a\frac{x^2}{2} = \frac{x^2}{4}.$

$1 \leq x \leq 2$ $\quad F(x) = \int_0^1 ax\,dx + \int_1^x a\,dx = \frac{1}{2}\left[\frac{1}{2} + (x-1)\right]$

$2 \leq x \leq 3$ $\quad F(x) = \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^x (-ax+3a)\,dx$

$$= -\frac{5}{4} + \frac{3x}{2} - \frac{x^2}{4}.$$

$x \geq 3$ $\quad F(x) = \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx + \int_3^x 0\,dx$

$$= 1$$

$$P(X \leq 2.5) = F(2.5) = -\frac{1}{4}(2.5)^2 + \frac{3}{2}(2.5) - \frac{5}{4}. = .9375.$$

(3) $a=5$

$E(x-5)=2$

$E(X)=7=\bar{x}$

$E(x-5)^2=20$

$E(x-\bar{x}+2)^2=20$

$E[(x-\bar{x})^2+4(x-\bar{x})+4]=20$

$\mu_2+4=20.$

$\mu_2=16.$

$E(x-\bar{x}+2)^3=40$

$\mu_3=-64.$

$E(x-\bar{x}+2)^4=50.$

$\mu_4=162.$

$\beta_1=\frac{\mu_3^2}{\mu_2^3}=\frac{4096}{4096}=1$

$\beta_2=\frac{\mu_4}{\mu_2^2}=\cdot 6328$

$\beta_1\neq 0$ curve is not symmetric

$\beta_2<3 \rightarrow$ platykurtic

(4) Here, the random variable x, denoting the no. of components functioning is Binomial Variate $x \sim B(n,p)$

It is Required to find p such that

$$\sum_{r=3}^{5} {}^5C_r p^5 q^{5-r} \geq \sum_{r=2}^{3} {}^3C_r p^r q^{3-r}$$

$\Rightarrow 10p^3q^2+5p^4q+p^5 \geq 3p^2q+p^3$

$\Rightarrow 10p^3(1-p)^2+5p^4(1-p) \geq 3p^2(1-p)+p^3.$

$\Rightarrow 3p^2(2p^3-5p^2+4p-1) \geq 0$

$(2p-1)\,3p^2(p-1)^2 \geq 0$

$p \geq 1/2$ Ans.

⑤ $M_X(t) = E(e^{tX})$

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx.$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\left\{(x-\mu)^2 + 2\sigma^2 tx\right\}\right] dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\left\{x^2+\mu^2 - 2x(\mu+\sigma^2 t) + \sigma^4 t^2 + 2\mu\sigma^2 t - \sigma^4 t^2 - 2\mu\sigma^2 t\right\}\right] dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\{x-(\mu+\sigma^2 t)\}^2\right] \cdot \exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right) dx$$

$$= e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$$\mu_1' = \left(\frac{d}{dt}(M_X(t))\right)_{t=0} = \left(e^{\mu t + \frac{1}{2}\sigma^2 t^2} \times (\mu + \sigma^2 t)\right) = \mu$$

$$\mu_2' = \left(\frac{d^2}{dt^2}(M_X(t))\right)_{t=0} = \sigma^2 + \mu^2$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= \sigma^2 + \mu^2 - \mu^2 = \sigma^2$$

or

$$E(X) = \sum x\, p(x)$$

$$= \sum x \cdot \frac{e^{-m} m^x}{x!}$$

$$= \sum \frac{e^{-m}\, m \cdot m^{x-1}}{(x-1)!}$$

$$= m \sum \frac{e^{-m} m^{x-1}}{(x-1)!}$$

$$= m \times 1 = m$$

$$E(X^2) = E(x(x-1)+x)$$

$$= \sum x(x-1)\frac{e^{-m}m^x}{x!} + \sum x \frac{e^{-m} m^x}{x!}$$

$$= m^2 + m.$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= m^2 + m - m^2$$

$$\mu_2 = m$$

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