Govt. Women Engineering College, Ajmer

B.Tech –IV Semester(I Mid Term Feb. 2018) Branch:- ECE

Time:1 hr Sub: Optimization Techniques Max Marks:20

All questions carry equal marks

Q.1 Solve the LPP by Simplex Method:

Min. $Z = x_1 + x_2$

s.t. $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

and $x_1, x_2 \geq 0$

Q.2 Solve the minimum assignment problem:

Jobs	Machines I	II	III
A	1	3	6
B	2	4	8
C	5	3	4
D	6	9	7

Q.3 Use revised simplex method to solve the LPP:

Max. $Z = x_1 + x_2$

s.t. $3x_1 + 2x_2 \leq 6$

$x_1 + 4x_2 \leq 4$

and $x_1, x_2 \geq 0$

OR

Q.3 Solve the transportation Problem and test it for optimality:

Factory	Warehouse I	II	III	IV	supply
F_1	4	6	8	13	50
F_2	13	11	10	8	70
F_3	14	4	10	13	30
F_4	9	11	13	8	50
Demand	25	35	105	20	

Q.4 Solve graphically:

Max. $Z = 5x_1 + 3x_2$

s.t. $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

and $x_1, x_2 \geq 0$

OR

Q.4 Write the Dual of the LPP:

Max. $Z_p = x_1 + 3x_2$

s.t. $3x_1 + 2x_2 \leq 6$

$3x_1 + x_2 = 4$

and $x_1, x_2 \geq 0$

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B. Tech IV Semester (I MID TERM)
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Sub:- OPTIMIZATION TECHNIQUES

Q1. Solve the LPP by Simplex Method

$$\text{Min } Z = x_1 + x_2$$

s.t. $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

and $x_1, x_2 \geq 0$

Solution :- first write the LPP into standard form

$$\text{Max } (-Z) = Z' = -x_1 - x_2 + 0x_3 + 0x_4$$

$$2x_1 + x_2 - x_3 = 4$$

$$x_1 + 7x_2 - x_4 = 7$$

$$x_1, x_2, x_3, x_4 \geq 0$$

where x_3 and x_4 are surplus variables

But we do not get Basis $\begin{bmatrix} 2 & 1 & -1 & 0 \\ 1 & 7 & 0 & -1 \end{bmatrix}$

so we introduce artificial variables

$$\text{Max } (-Z) = -x_1 - x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$$

$$2x_1 + x_2 - x_3 + x_5 = 4$$

$$x_1 + 7x_2 - x_4 + x_6 = 7$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

where x_5 and x_6 are artificial variables

$$A = \begin{bmatrix} 2 & 1 & -1 & 0 & 1 & 0 \\ 1 & 7 & 0 & -1 & 0 & 1 \end{bmatrix} = [\alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4 \ \alpha_5 \ \alpha_6]$$

Now we construct first simplex table

			c_j	-1	-1	0	0	-M	-M
C_B	B	X_B	b	y_1	y_2	y_3	y_4	y_5	y_6
-M	α_5	x_5	4	2	1	-1	0	1	0
-M	α_6	x_6	7	1	$\boxed{7}$	0	-1	0	1 →
$z_j - c_j$				-3M +1	-8M +1 ↑	M	M	0	0

∴ all $z_j - c_j \ngeq 0$ ∴ we find entering vector as most negative $z_j - c_j$ is $-8M+1$ ie $z_2 - c_2$ therefore x_2 will be entering vector

for key element $= \min\left[\frac{4}{1}, \frac{7}{7}\right]$

$= \frac{7}{7}$

∴ $y_{22} = 7$ is key element and x_6 is departing vector, we drop the coloumn of x_6 in next simplex table

			c_j	-1	-1	0	0	-M
C_B	B	X_B	b	y_1	y_2	y_3	y_4	y_5
-M	α_5	x_5	3	$\boxed{13/7}$	0	-1	1/7	1 →
-1	α_2	x_2	1	1/7	1	0	-1/7	0
$z_j - c_j$				$-\frac{13}{7}M + 6/7$ ↑	0	M	$-\frac{1}{7}M + \frac{1}{7}$	0

since $z_1 - c_1$ is most negative ∴ x_1 will be entering vector

key element $= \min\left[\frac{21}{13}, \frac{7}{1}\right]$

$= \frac{21}{13}$

∴ $y_{11} = \frac{13}{7}$ is key element and x_5 is departing vector, we drop the coloumn of x_5 in next simplex table

			c_j	-1	-1	0	0
c_B	B	x_B	b	y_1	y_2	y_3	y_4
-1	α_1	x_1	21/13	1	0	-7/13	1/13
-1	α_2	x_2	10/13	0	1	1/13	-2/13
$z_j - c_j$				0	0	6/13	1/13

$\therefore$ all $z_j - c_j \geq 0$ $\therefore$ it is optimum level

$x_1 = \frac{21}{13}$, $x_2 = \frac{10}{13}$

$\text{Max}(-z) = -\frac{31}{13}$

$\text{Min } z = \frac{31}{13}$

Q2 Solve the minimum assignment Problem

	I	II	III
A	1	3	6
B	2	4	8
C	5	3	4
D	6	9	7

Solution:- first construct a square matrix by introducing a dummy coloumn to convert the Problem into a blanced Problem

	I	II	III	IV
A	1	3	6	0
B	2	4	8	0
C	5	3	4	0
D	6	9	7	0

Subtract least element of each row from every element of the corresponding row

	I	II	III	IV
A	1	3	6	0
B	2	4	8	0
C	5	3	4	0
D	6	9	7	0

Subtract least element of each coloumn from every element of the corresponding coloumn

	I	II	III	IV
A	0	0	2	0
B	1	1	4	0
C	4	0	0	0
D	5	6	3	0

Now apply assignment Procedure

	I	II	III	IV
A	$\boxed{0}$	~~0~~	2	~~0~~
B	1	1	4	$\boxed{0}$
C	4	$\boxed{0}$	~~0~~	~~0~~
D	5	6	3	~~0~~

above solution is not optimum, as fourth row has no assignment, now we draw straight lines

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	I	II	III	IV
A	$\boxed{0}$	$\cancel{0}$	2	$\cancel{0}$
B	1	1	4	$\boxed{0}$
C	4	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
D	5	6	3	$\cancel{0}$

least uncovered element is 1, so we subtract 1 from uncovering elements and add to the elements which lies at the intersection of straight lines and again apply assignment Procedure

	I	II	III	IV
A	$\boxed{0}$	$\cancel{0}$	2	1
B	$\cancel{0}$	$\boxed{0}$	3	$\cancel{0}$
C	4	$\cancel{0}$	$\boxed{0}$	1
D	4	5	2	$\boxed{0}$

optimal assignment is

A → I , B → II , C → III , D → IV

minimum cost of assignment is $1+4+4+0 = 9$ units

Q3 Use revised simplex method to solve LPP

$$\text{Max. } Z = x_1 + x_2$$

$$3x_1 + 2x_2 \le 6$$

$$x_1 + 4x_2 \le 4$$

and $x_1, x_2 \geq 0$

Solution First write the LPP into standard form

$$\text{Max. } z = x_1 + x_2 + 0x_3 + 0x_4$$

$$3x_1 + 2x_2 + x_3 = 6$$

$$x_1 + 4x_2 + x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

where x_3 and x_4 are slack variables
Basic variables are x_3 and x_4

Now $\hat{A} = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix}$, $\hat{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\hat{B}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\hat{b} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$

Now calculate Net evaluation

$$z_j - c_j = (c_B B^{-1}\ 1)\hat{A} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & 0 & 0 \end{bmatrix}$$

Since $z_1 - c_1$ and $z_2 - c_2$ both are most negative so any one can be entering vector. Let x_1 is entering vector

$$\hat{x}_1 = \hat{B}^{-1}\hat{a}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

$$\hat{x}_B = \hat{B}^{-1}\hat{b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

Now first revised simplex table is

	$\hat{B}^{-1}$			$\hat{x}_B$	$\hat{x}_1$	Mini ratio
x_3	1	0	0	6	$\boxed{3}$	$6/3 = 2$ →
x_4	0	1	0	4	1	$4/1 = 4$
	0	0	1	0	-1	

$x_{11} = 3$ is key element and x_3 is departing vector

Now new Basis is

$$\hat{B}^{-1} = \begin{array}{c} x_1 \\ x_4 \\ \\ \end{array} \begin{bmatrix} 1/3 & 0 & 0 \\ -1/3 & 1 & 0 \\ 1/3 & 0 & 1 \end{bmatrix}$$

Net evaluation is $z_j - c_j = \begin{bmatrix} 1/3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix}$

$$= \left[0, \frac{-1}{3}, \frac{1}{3}, 0\right]$$

Since $z_2 - c_2$ is most negative therefore x_2 will be entering vector

$$\hat{x}_2 = \hat{B}^{-1} \hat{a}_2 \cdot \begin{bmatrix} 1/3 & 0 & 0 \\ -1/3 & 1 & 0 \\ 1/3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 10/3 \\ -1/3 \end{bmatrix}$$

$$\hat{x}_B = \hat{B}^{-1} \hat{b} = \begin{bmatrix} 1/3 & 0 & 0 \\ -1/3 & 1 & 0 \\ 1/3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

Second revised simplex table is

	$\hat{B}^{-1}$			$\hat{x}_B$	$\hat{x}_2$	Mini ratio
x_1	$\frac{1}{3}$	0	0	2	$2/3$	$6/2 = 3$
x_4	$-1/3$	1	0	2	$\boxed{10/3}$	$6/10$ →
	$1/3$	0	1	2	$-1/3$	

x_4 is departing vector, New Basis is

$$\hat{B}^{-1} = \begin{array}{c} x_1 \\ x_2 \\ \\ \end{array} \begin{bmatrix} 2/5 & -1/5 & 0 \\ -1/10 & 3/10 & 0 \\ 3/10 & 1/10 & 1 \end{bmatrix}$$

Net evaluation $= \left[\frac{3}{10} \quad \frac{1}{10} \quad 1\right] \begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix}$

$$= \left[0 \quad 0 \quad \frac{3}{10} \quad \frac{1}{10}\right]$$

$\therefore$ all $z_j - c_j \geq 0$ $\therefore$ it is optimum solution

$$\hat{x}_B = \hat{B}^{-1}\hat{b} = \begin{bmatrix} \frac{2}{5} & \frac{-1}{5} & 0 \\ \frac{-1}{10} & \frac{3}{10} & 0 \\ \frac{3}{10} & \frac{1}{10} & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 8/5 \\ 3/5 \\ 11/5 \end{bmatrix}$$

$x_1 = \frac{8}{5}$, $x_2 = \frac{3}{5}$, $\text{Max } z = \frac{11}{5}$

Q3 Solve the transportation Problem and test it for optimality

	W_1	W_2	W_3	W_4	
F_1	4	6	8	13	50
F_2	13	11	10	8	70
F_3	14	4	10	13	30
F_4	9	11	13	8	50
	25	35	105	20	200 / 185

Solution :- first convert the Problem to a balanced Problem and find initial solution using vogel's method

						I	II	III	IV	V	VI	VII
(25) 4	(5) 6	(20) 8	13	0	~~50~~ ~~25~~ ~~20~~ 0	4	6	2	2	5		
13	11	(35) 10	(20) 8	(15) 0	~~70~~ ~~55~~ ~~35~~ 0	8	8	2	2	2	2	10
14	(30) 4	10	13	0	30 0	4	4	6				
9	11	(50) 13	8	0	50 0	8	8	3	3	5	5	13
~~25~~ 0	~~35~~ ~~5~~ 0	~~105~~ ~~85~~ (50) 0	~~20~~ 0	~~15~~ 0								

I	5	2	2	5	0
II		2	2	5	0
III		2	2	5	
IV		5	2	5	
V			2	5	
VI			3	8	
VII			3	8	

Now no. of allocated cells are
$m+n-1 = 4+5-1 = 8$

Test for optimality

	$v_1=4$	$v_2=6$	$v_3=8$	$v_4=6$	$v_5=-2$
$u_1=0$	(25) 4	(5) 6	(20) 8	13 [7]	0 [2]
$u_2=2$	13 [7]	11 [3]	(35) 10	(20) 8	(15) 0
$u_3=-2$	14 [12]	(30) 4	10 [4]	13 [9]	0 [4]
$u_4=5$	9 [0]	11 [0]	(50) 13	8 [-3]	0 [-3]

Construct loop from most negative d_{ij} but two d_{ij}'s are most negative, so we can construct loop from anyone

(25) 4	(5) 6	(20) 8	13 [7]	0 [2]
13 [7]	11 [3]	(35) 10	(20) 8	(15) 0
14 [12]	(30) 4	10 [4]	13 [9]	0 [4]
9 [0]	11 [0]	(50) 13	8 [-3]	0 [-3]

$20+0 = 20$
$20-20 = 0$
$20+35 = 55$
$20-50 = 30$

the minimum allocation at -ve sign is 20, Thus 20 is added to the allocation at +ve sign and sustracted from allocations at -ve sign to get new BFS.

(25) 4	(5) 6	(20) 8	13	0
13	11	(55) 10	8	(15) 0
14	(30) 4	10	13	0
9	11	(30) 13	(20) 8	0

Test for optimality

	$v_1=4$	$v_2=6$	$v_3=8$	$v_4=3$	$v_5=-2$
$u_1=0$	25 (4)	5 (6)	20 (8)	[10] (13)	[2] (0)
$u_2=2$	[7] (13)	[3] (11)	55 (10)	[3] (8)	15 (0)
$u_3=2$	[12] (14)	30 (4)	[4] (10)	[12] (13)	[4] (0)
$u_4=5$	[0] (9)	[0] (11)	30 (13)	20 (8)	[-3] (0)

$\theta + 15 = 15$

$15 - 15 = 0$

$55 + 15 = 70$

$30 - 15 = 15$

construct loop from most negative d_{ij}, the minimum allocation at -ve sign is 15, so fifteen is added to allocations at +ve sign and subtracting to allocation at -ve sign to get new BFS, now test for optimality of new solution

	$v_1=4$	$v_2=6$	$v_3=8$	$v_4=3$	$v_5=-5$
$u_1=0$	25 (4)	5 (6)	20 (8)	[10] (13)	[5] (0)
$u_2=2$	[7] (13)	[3] (11)	70 (10)	[3] (8)	[3] (0)
$u_3=2$	[8] (14)	30 (4)	[0] (10)	[8] (13)	[3] (0)
$u_4=5$	[0] (9)	[0] (11)	15 (13)	20 (8)	15 (0)

$\therefore$ all $d_{ij} \geq 0$ $\therefore$ it is optimum solution

$x_{11}=25$, $x_{12}=5$, $x_{13}=20$, $x_{32}=30$ $x_{23}=70$, $x_{43}=15$

$x_{44}=20$, $x_{45}=15$

cost of transportation is $4\times25 + 6\times5 + 20\times8 + 70\times10 + 30\times4 + 15\times3 + 20\times8 + 15\times0 = $ Rs. 1465

Q4 Solve graphically

Max. $Z = 5x_1 + 3x_2$

s.t. $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$

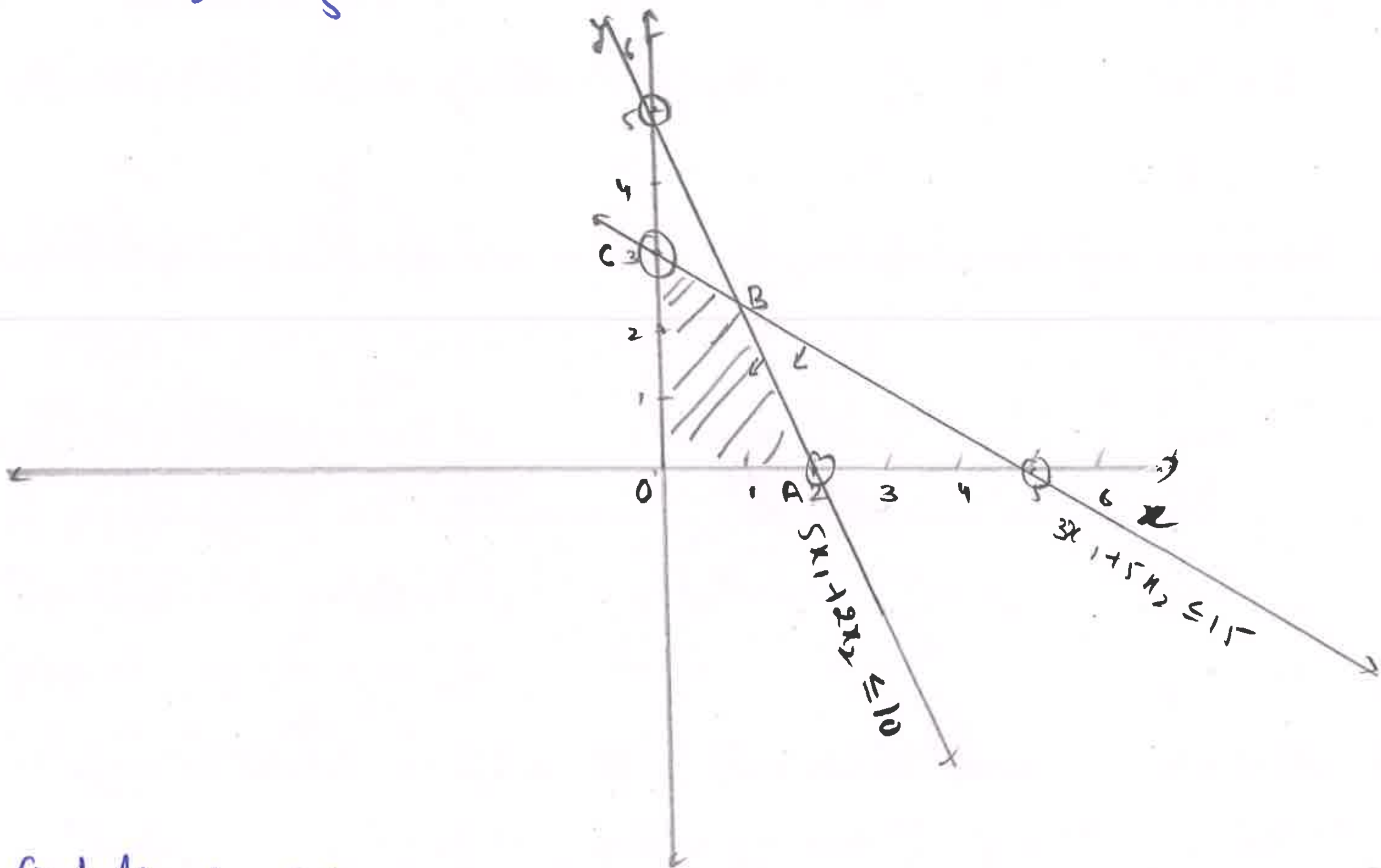
Solution:- Draw straight corresponding to each constraint

$3x_1 + 5x_2 = 15$

$$\frac{x_1}{5} + \frac{x_2}{3} = 1$$

and $5x_1 + 2x_2 = 10$

$$\frac{x_1}{2} + \frac{x_2}{5} = 1$$



find the feasible region corresponding to each constraint and then common region. find the value of objective function at corners of common region

O	(0 0)	$Z = 0$
A	(2 0)	$Z = 10$
B	$(20/19, 45/19)$	$Z = 235/19 \simeq 12.37$
C	(0,3)	$Z = 9$

∴ optimum solution is

$$x_1 = \frac{20}{19}, \quad x_2 = \frac{45}{19} \quad \text{and} \quad Z = \frac{235}{19}$$

Q4 Write the dual of the LPP

$$\text{Max. } Z_P = x_1 + 3x_2$$
$$3x_1 + 2x_2 \leq 6$$
$$3x_1 + x_2 = 4$$
$$x_1, x_2 \geq 0$$

Solution:- First convert the given Primal into standard form

$$\text{Max. } Z_P = CX$$
$$AX \leq b$$
$$X \geq 0$$

$$\Rightarrow \text{Max. } Z_P = x_1 + 3x_2$$
$$3x_1 + 2x_2 \leq 6$$
$$3x_1 + x_2 \leq 4$$
$$3x_1 + x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

$$\Rightarrow \text{Max. } Z_P = x_1 + 3x_2$$
$$3x_1 + 2x_2 \leq 6$$
$$3x_1 + x_2 \leq 4$$
$$-3x_1 - x_2 \leq -4$$
$$x_1, x_2 \geq 0$$

Now Primal is in standard form, so we can write dual as

$$\text{Min } Z_D = b^T W$$
$$A^T W \geq C^T$$
$$W \geq 0$$

$\Rightarrow$ $\text{Min} \cdot Z_D = 6w_1 + 4w_2 - 4w_3$

$3w_1 + 3w_2 - 3w_3 \geq 1$

$2w_1 + w_2 - w_3 \geq 3$

$w_1, w_2, w_3 \geq 0$

Let $w_2 - w_3 = w_2'$

$\Rightarrow$ $\text{Min } Z_D = 6w_1 + 4w_2'$

$3w_1 + 3w_2' \geq 1$

$2w_1 + w_2' \geq 3$

$w_1 \geq 0$ and w_2' is unrestricted in sign