Govt. Women Engineering College, Ajmer

B.Tech –II Semester (I Mid Term March. 2018) Branch: EE,ECE & ME

Time:1 hr **Sub: Engineering Mathematics -II** **Max Marks:20**

All questions carry equal marks

Q.1(i) Investigate the values of a and b so that the equations

x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + az = b

have (i) no solution (ii) unique solution (iii) infinite number of solutions.

(ii) State Cayley- Hamilton Theorem

Q.2 (i) Find the rank of the matrix $\begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -1 \\ -1 & 2 & 7 \end{bmatrix}$

(ii) Define rank of a matrix

Q.3 Solve $\frac{dy}{dx} = \frac{1}{xy(x^2y^2+1)}$

Q.4 (i) write order and degree of the differential equation

$$\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^3 + 3x^2y = x^4$$

(ii) Solve $\frac{d^2y}{dx^2} - 4y = e^x + sinx$

Q.5 Solve $(D^2 - 1)y = x^2cosx$

OR

Q.5 Solve $(x^3e^x - my^2)dx + mxy\,dy = 0$

Q1. (i) Investigate the values of a and b so that the equations

$x+2y+3z=6, \quad x+3y+5z=9, \quad 2x+5y+9z=b$

has (i) no solution (ii) unique solution (iii) infinite numbers of solutions

Solution Here

$$C = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 1 & 3 & 5 & 9 \\ 2 & 5 & a & b \end{array}\right]$$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - 2R_1$

$$C \cdot \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & a-6 & b-12 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$C \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & a-8 & b-15 \end{array}\right]$$

Case (i) it is possible when $\rho(A) \neq \rho(C)$

$\Rightarrow \quad a-8=0$ and $b-15 \neq 0$

$\Rightarrow \quad \rho(A)=2$ and $\rho(C)=3$

$\therefore \quad \rho(A) \neq \rho(C)$

$\therefore$ system has no solution

case (ii) for this $\rho(A) = \rho(C) = 3$ (number of variables)

$\Rightarrow a - 8 \neq 0$ and b can have any value

$\Rightarrow \rho(A) = 3 = \rho(C)$

case (iii) for this $\rho(A) = \rho(C) < 3$

$\Rightarrow a - 8 = 0$ and $b - 15 = 0 \Rightarrow a = 8,\ b = 15$

$\Rightarrow \rho(A) = \rho(C) = 2$

Q1 (ii) Every square matrix A satisfies its own characteristic equation

i.e. $|A - \lambda I_n| = (-1)^n \lambda^n + P_1 \lambda^{n-1} + P_2 \lambda^{n-2} + \cdots + \lambda P_{n-1} + P_n = 0$

$\Rightarrow (-1)^n A^n + P_1 A^{n-1} + P_2 A^{n-2} + \cdots + P_{n-1} A + P_n I_n = 0$

Q2 (i) find the rank of the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -1 \\ -1 & 2 & 7 \end{bmatrix}$$

Solution

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 + R_1$

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 10 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

Q2(ii) A matrix A $(m \times n)$ is said to be of rank 'r' if

(i) it has at least one non-zero minor of order 'r' and

(ii) all other minors of order greater than r, if any, are zero.

Q3. Solve

$$\frac{dy}{dx} = \frac{1}{xy(x^2y^2+1)}$$

$$\Rightarrow \frac{dx}{dy} = x^3y^3 + xy$$

$$\Rightarrow \frac{dx}{dy} - yx = y^3x^3$$

$$\Rightarrow x^{-3}\frac{dx}{dy} - x^{-2}y = y^3$$

Let $x^{-2} = t$

$$-2x^{-3}\frac{dx}{dy} = \frac{dt}{dy}$$

$$\Rightarrow -\frac{1}{2}\frac{dt}{dy} - ty = y^3$$

$$\Rightarrow \frac{dt}{dy} + 2ty = -2y^3$$

$$\Rightarrow \quad IF \; e^{\int 2y\,dy} = e^{y^2}$$

∴ Solution is

$$te^{y^2} = \int -2y^3 e^{y^2} dy + c$$

Let $y^2 = v$

$2y\,dy = dv$

$$te^{y^2} = -\int ve^{v} dv + c$$

$$te^{y^2} = -(v-1)e^{v} + c$$

$\therefore\ t = x^2$ & $u = y^2$

$$\frac{e^{y^2}}{x^2} = -(y^2-1)e^{y^2} + C$$

Q 4 (i) write the order and degree of the differential equation

$$\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^3 + 3x^2y = x^4$$

solution:- Order – 2

degree – 1

Q 4 (ii) Solve :- $\frac{d^2y}{dx^2} - 4y = e^x + \sin x$

solution:- Let $\frac{d}{dx} = D$

$$(D^2-4)y = e^x + \sin x$$

A.E. is $m^2-4=0$

$m = \pm 2$

CF is $c_1e^{2x} + c_2e^{-2x}$

PI $\frac{1}{D^2-4}(e^x + \sin x)$

$$\Rightarrow \frac{1}{D^2-4}e^x + \frac{1}{D^2-4}\sin x$$

$$\frac{1}{1^2-4}e^x + \frac{1}{(-1)-4}\sin x$$

$$\frac{1}{-3}e^x - \frac{1}{5}\sin x$$

complete solution is $y = c_1e^{2x} + c_2e^{-2x} - \frac{1}{3}e^x - \frac{1}{5}\sin x$

Q5 Solve

$$(D^2-1)y = x^2\cos x$$

Solution:- A.E. is $m^2-1=0$

$m = \pm 1$

CF is $c_1 e^x + c_2 e^{-x}$

PI is $\frac{1}{D^2-1} x^2 \cos x$

$$\Rightarrow \text{Real Part of} \left\{ \frac{1}{D^2-1} x^2 e^{ix} \right\}$$

$$\Rightarrow \text{Real Part of} \left\{ e^{ix} \frac{1}{(D+i)^2-1} x^2 \right\}$$

$$\Rightarrow \text{Real Part of} \left\{ e^{ix} \frac{1}{D^2-1+2iD-1} x^2 \right\}$$

$$\Rightarrow \text{Real part of} \left\{ e^{ix} \frac{1}{D^2+2iD-2} x^2 \right\}$$

$$\Rightarrow \text{Real part of} \left\{ e^{ix} \frac{1}{-2\left[1-\left(\frac{D^2+2iD}{2}\right)\right]} x^2 \right\}$$

$$\Rightarrow \text{Real Part of} \left\{ \frac{e^{ix}}{-2} \left[1-\left(\frac{D^2+2iD}{2}\right)\right]^{-1} x^2 \right\}$$

$$\Rightarrow \text{Real Part of} \left\{ \frac{e^{ix}}{-2} \left[1+\left(\frac{D^2+2iD}{2}\right)+\left(\frac{D^2+2iD}{2}\right)^2+\cdots\right] x^2 \right\}$$

$$\Rightarrow \text{Real Part of} \left\{ \frac{e^{ix}}{-2} \left[x^2 + \left(\frac{D^2+2iD}{2}\right)x^2 + \left(\frac{D^4-4D^2+4iD^3}{4}\right)x^2 + \cdots\right] \right\}$$

$$\Rightarrow \text{Real Part of} \left\{ \frac{e^{ix}}{-2} \left[x^2 + \left(\frac{2+4ix}{2}\right) + \left(\frac{-8}{4}\right)\right] \right\}$$

$\Rightarrow$ Real Part of $\left\{ \frac{e^{ix}}{-2}\left[x^2+1+2ix-2\right]\right\}$

$\Rightarrow$ Real Part of $\left\{ \frac{e^{ix}}{-2}\left[x^2+2ix-1\right]\right\}$

$\Rightarrow$ Real part of $\left\{ \frac{(\cos x+i\sin x)}{-2}(x^2+2ix-1)\right\}$

$$= \frac{-x^2\cos x}{2} + \frac{\cos x}{2} + x\sin x$$

$$\therefore y = c_1e^{x} + c_2e^{-x} + x\sin x + \frac{1}{2}(1-x^2)\cos x$$

Q5 Solve

$$(x^3e^{x} - my^2)\,dx + mxy\,dy = 0$$

Solution:- Here $M = x^3e^{x} - my^2$, $N = mxy$

Now $\frac{\partial M}{\partial y} = -2my$ $\qquad \frac{\partial N}{\partial x} = my$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ eqⁿ is not an exact equation

Now

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$$

$$= \frac{1}{mxy}(-2my - my) = \frac{-3my}{mxy} = -\frac{3}{x} = f(x)$$

I.F is $e^{\int f(x)dx} = e^{\int \frac{-3}{x}dx} = e^{-3\log x} = \frac{1}{x^3}$

Now after multiplying by I.F equation becomes

$$\left(e^{x} - \frac{my^2}{x^3}\right)dx + \frac{my}{x^2}\,dy = 0$$

Now New M and N are

$$M = e^x - \frac{my^2}{x^3}, \quad N = \frac{my}{x^2}$$

$$\frac{\partial M}{\partial y} = \frac{-2my}{x^3}, \quad \frac{\partial N}{\partial x} = -\frac{2my}{x^3}$$

$$\therefore \frac{\partial M}{\partial y} \quad \frac{\partial N}{\partial x}$$

$\therefore$ Now new equation is an exact equation

$$\therefore U = \int M dx = \int \left(e^x - \frac{my^2}{x^3}\right) dx = e^x + \frac{my^2}{2x^2}$$

now $\frac{\partial U}{\partial y} = 0 + \frac{2my}{2x^2} = \frac{my}{x^2}$

Now $N - \frac{\partial U}{\partial y} = \frac{my}{x^2} - \frac{my}{x^2} = 0$

$$\Rightarrow V = \int \left(N - \frac{\partial U}{\partial y}\right) dy = \int 0 \, dy = C_1$$

Solution is

$$U + V = C$$

$$\Rightarrow e^x + \frac{my^2}{2x^2} + C_1 = C$$

$$\Rightarrow e^x + \frac{my^2}{2x^2} = K \quad \text{where } K = C - C_1$$